

# A Supersymmetric Model with the Gauge Symmetry

$$SU(3)_1 \times SU(2)_1 \times U(1)_1 \times SU(3)_2 \times SU(2)_2 \times U(1)_2$$

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## Abstract

A supersymmetric model with two copies of the Standard Model gauge groups is constructed in the gauge mediated supersymmetry breaking scenario. The supersymmetry breaking messengers are in a simple form. The Standard Model is obtained after first step gauge symmetry breaking. In the case of one copy of the gauge interactions being strong, a scenario of electroweak symmetry breaking is discussed, and the gauginos are generally predicted to be heavier than the sfermions.

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The naturalness of the Standard Model (SM) implies new physics at the TeV energy scale [1]. The most attractive new physics would be dynamical electro-weak symmetry breaking (EWSB), as for example the technicolor scenario [2] if it did not have serious flavor changing neutral current (FCNC) problems. Furthermore, the heavy top quark needs the assistance of the top-color mechanism [3] in this scenario. Both technicolor and top-color ideas introduce new gauge interactions which are strong at the TeV scale. New hierarchy problems may arise because realistic models in this framework introduce scalar fields.

Another beautiful new physics scenario is supersymmetry (SUSY) [4] which is broken dynamically [5]. It makes the grand unification theories (GUTs) [6] viable. There are indirect experimental evidences for GUTs from LEP and neutrino physics. The SUSY extension of the SM still suffers from certain problems [7]. It has been realized that SUSY breaking should occur in a hidden sector [8]. It was thus very simple to take gravity as the interaction which mediates SUSY breaking. However, in general the supergravity [7] case has FCNC problems. This problem can be avoided, if the energy scale of messenger physics is much lower than the Planck scale. Then it is simple to use gauge interactions to mediate SUSY breaking [9,10], with considerably low scales of SUSY breaking and messenger masses. However, this gauge mediated SUSY breaking (GMSB) suffers seriously from the so-called  $\mu$ -problem [10,11].

Nature might be more complicated than we thought. In this paper, we consider a SUSY model which has two copies of the SM gauge groups. In addition to the above-mentioned difficulties in the new physics approaches, we especially note that the fermion mass pattern and CP violation have no full understanding. Ref. [12] proposed that SUSY might be used for an understanding of the flavor puzzle: the muon mass originates from the sneutrino vacuum expectation values (VEVs), whereas the tau mass originates from the Higgs VEV. To be consistent, later it was proposed [13] that the top quark obtains its mass mostly from some dynamical mechanism, namely the top-color mechanism. Furthermore this SUSY top-color model with GMSB is well-motivated since it has no FCNC problem. But it has some drawbacks. It is irrelevant to GUTs. And the SUSY breaking messengers are in a very complicated form.

It will be interesting to consider models with  $SU(3) \times SU(2) \times U(1) \times G$  gauge interactions, where  $G$  stands for an unspecified group. The SUSY breaking messengers are taken to be fundamental representations of  $SU(3) \times SU(2) \times U(1)$  and  $G$ . To be specific and without losing generality, in this paper we add one more  $SU(2)$  gauge interaction into the top-color like interactions. The gauge interactions are separately unifiable. The form of SUSY breaking messengers is relatively simple. We do not consider the fermion mass problem. One group of gauge interactions is not necessarily strong. There are other motivations for such theories [14–18].

We study a SUSY theory with the gauge group  $G_1 \times G_2$  in the framework of GMSB, where  $G_i = SU(3)_i \times SU(2)_i \times U(1)_i$  ( $i = 1, 2$ ). The three coupling constants of  $G_1$  can be large, and those of  $G_2$  are small at the TeV scale. The three generations of matter carry nontrivial quantum numbers of  $G_2$  only. These numbers are assigned in the same way as they are under the SM gauge group.

Let us first discuss SUSY breaking. One gauge singlet chiral superfield  $X$  is introduced for this purpose with the following superpotential,

$$\mathcal{W}_0 = -\mu_{\text{SUSY}}^2 X, \quad (1)$$

where  $\mu_{\text{SUSY}}$  is the SUSY breaking scale. The SUSY breaking is communicated to the observable sector through the gauge interactions by the messengers with  $SU(3)_1 \times SU(2)_1 \times U(1)_1 \times SU(3)_2 \times SU(2)_2 \times U(1)_2$  quantum numbers

$$\begin{aligned} T_1, T_1' &= (3, 2, \frac{1}{3}, 1, 1, 0), & \bar{T}_1, \bar{T}_1' &= (\bar{3}, 2, -\frac{1}{3}, 1, 1, 0); \\ T_2, T_2' &= (1, 1, 0, 3, 2, \frac{1}{3}), & \bar{T}_2, \bar{T}_2' &= (1, 1, 0, \bar{3}, 2, -\frac{1}{3}) \end{aligned} \quad (2)$$

which have direct interactions with  $X$ . The relevant superpotential is

$$\begin{aligned} \mathcal{W}_1 &= m_1(\bar{T}_1' T_1 + T_1' \bar{T}_1) + m_2 T_1 \bar{T}_1 + m_3(\bar{T}_2' T_2 + T_2' \bar{T}_2) + m_4 T_2 \bar{T}_2 \\ &+ X(c_1 T_1 \bar{T}_1 + c_2 T_2 \bar{T}_2), \end{aligned} \quad (3)$$

where  $c_1$  and  $c_2$  are coupling constants of order one,  $m_j$  ( $j = 1 - 4$ ) are mass parameters of the same order. It is required that  $m_2/m_4 \neq c_1/c_2$  so that the terms proportional to

$m_2$  and  $m_4$  cannot be eliminated by a shift in  $X$ . The model conserves the number of the messengers. In addition, the superpotential has a discrete symmetry of exchanging  $T_i^{(\prime)}$  and  $\bar{T}_i^{(\prime)}$ . The introduction of SUSY breaking is a generalization of that given in Ref. [9].

The messenger fields are massive at tree level. Because the auxiliary component of the  $X$  field has non-vanishing VEV  $\mu_{\text{SUSY}}^2$ , SUSY breaking occurs in the fields  $T_i^{(\prime)}$ 's and  $\bar{T}_i^{(\prime)}$ 's at tree level.

It is via quantum effects that the messengers mediate SUSY breaking to the  $G_1$  and  $G_2$  sector. In the case of weak gauge interactions, the perturbation method based on the gauge coupling constant expansion is used to calculate soft SUSY breaking masses. Gauginos acquire masses mainly at one-loop order [9,10],

$$M_{\lambda_r^{(\prime)}} \simeq \frac{\alpha_r^{(\prime)}}{4\pi} c_1 \frac{\mu_{\text{SUSY}}^2}{m_1}, \quad (4)$$

where  $\alpha_r^{(\prime)} = g_r^{(\prime)2}/4\pi$  with  $g_r^{(\prime)}$  being the gauge coupling constants of  $G_1$  ( $G_2$ ). And  $r = 1, 2, 3$  corresponding to the groups  $U(1)$ ,  $SU(2)$ , and  $SU(3)$ , respectively. The scalar particles of the matter fields in  $G_1$  and  $G_2$  obtain soft masses at two-loop order (except for the messengers).

In case  $G_1$  is strong, the corresponding soft masses cannot be calculated perturbatively. They should be the order of

$$M_{\lambda_r} \simeq c_1 \frac{\mu_{\text{SUSY}}^2}{m_1}. \quad (5)$$

There might be a suppression factor which ranges  $1 - 1/10$ , because nevertheless there is no tree-level interaction between these matter and  $X$ .

The  $G_1 \times G_2$  gauge symmetries break down spontaneously to the SM,  $SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_c$ ,  $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$  and  $U(1)_1 \times U(1)_2 \rightarrow U(1)_Y$  through a pair of Higgs superfields which are nontrivial under both  $G_1$  and  $G_2$ . Their  $SU(3)_1 \times SU(3)_2 \times SU(2)_1 \times SU(2)_2 \times U(1)_1 \times U(1)_2$  quantum numbers are assigned as follows,

$$\Phi_1(3, \bar{3}, 2, 2, \frac{1}{3}, -\frac{1}{3}), \quad \Phi_2(\bar{3}, 3, 2, 2, -\frac{1}{3}, \frac{1}{3}). \quad (6)$$

Their scalar components develop VEVs. One gauge singlet superfield  $Y$  is introduced for the gauge symmetry breaking. The superpotential of them is written as follows,

$$\mathcal{W}_2 = c'Y[\text{Tr}(\Phi_1\Phi_2) - \mu'^2], \quad (7)$$

where the trace is taken with regard to both  $SU(3)_1 \times SU(3)_2$  and  $SU(2)_1 \times SU(2)_2$ .  $\mu'$  is the energy scale relevant to the gauge symmetry breaking, and  $c'$  is the coupling constant. The way of introducing  $Y$  and  $X$  more naturally was discussed in Ref. [19] where this kind of field was taken to be composite. Note that the  $\Phi_i$ 's have no direct interaction with the field  $X$ . They get soft masses

$$m_{\Phi_1}^2 = m_{\Phi_2}^2 = m_\Phi^2, \quad (8)$$

where

$$m_\Phi^2 \simeq \frac{1}{(4\pi)^2} \sum_r (\alpha_r^2 + \alpha_r'^2) \left( c_1 \frac{\mu_{\text{SUSY}}^2}{m_1} \right)^2 \quad (9)$$

in the weak interaction case. In the strong interaction case  $m_\Phi$  is that given in Eq. (5) .

The VEVs of the  $\Phi_i$  are written as

$$\langle \Phi_{1s} \rangle = v_1 I_3 \otimes I_2, \quad \text{and} \quad \langle \Phi_{2s} \rangle = v_2 I_3 \otimes I_2, \quad (10)$$

where  $I_3$  and  $I_2$  are the unit matrices in the space of  $SU(3)_1 \times SU(3)_2$  and  $SU(2)_1 \times SU(2)_2$ , respectively.  $v_1$  and  $v_2$  are determined by the minimum of the following scalar potential:

$$V = |c'(3v_1v_2 - \mu'^2)|^2 + \frac{g_1^2 + g_1'^2}{2}(v_1^2 - v_2^2)^2 + m_\Phi^2(v_1^2 + v_2^2). \quad (11)$$

It is easy to see that for  $c'\mu'^2 \geq m_\Phi^2$ ,

$$v_1 = v_2 = \frac{1}{\sqrt{3}} \left( \mu'^2 - \frac{m_\Phi^2}{c'} \right)^{1/2}. \quad (12)$$

The coupling constants of the SM  $SU(3)_c \times SU(2)_L \times U(1)_Y$  are

$$\frac{1}{g_s^2} = \frac{1}{g_3^2} + \frac{1}{g_3'^2}, \quad \frac{1}{g^2} = \frac{1}{g_2^2} + \frac{1}{g_2'^2}, \quad \frac{1}{g'^2} = \frac{1}{g_1^2} + \frac{1}{g_1'^2}. \quad (13)$$

It is also easy to show that, orthogonal to the massless fields, the massive gauge bosons are

$$\tilde{A}_r = \frac{g_r A_r + g_r' A_r'}{\sqrt{g_r^2 + g_r'^2}}, \quad \text{with masses} \quad m_r = \sqrt{g_r^2 + g_r'^2} \sqrt{v_1^2 + v_2^2}. \quad (14)$$

Note in the above expression,  $r$  is not summed.

The full gaugino masses are determined by both the soft masses and the spontaneous gauge symmetry breaking. The gauge interactions of the Higgs fields  $\Phi_{1,2}$  are given by

$$\begin{aligned} \mathcal{L} = & \text{Tr} \left( \Phi_1^\dagger e^{2g_r V_r} \Phi_1 e^{2g'_r V'_r} + \Phi_2^\dagger e^{-2g_r V_r} \Phi_2 e^{-2g'_r V'_r} \right) |_{\theta\theta\bar{\theta}\bar{\theta}} \\ & \supset \sqrt{2}ig_r \text{Tr} \left( \Phi_{1_s}^* \lambda_r \psi_1 + \psi_1^\dagger \lambda_r^\dagger \Phi_{1_s} - \Phi_{2_s}^* \lambda_r \psi_2 - \psi_2^\dagger \lambda_r^\dagger \Phi_{2_s} \right) \\ & + \sqrt{2}ig'_r \text{Tr} \left( \psi_1 \lambda'_r \Phi_{1_s}^* + \Phi_{1_s} \lambda'^{\dagger}_r \psi_1^\dagger - \psi_2 \lambda'_r \Phi_{2_s}^* - \Phi_{2_s} \lambda'^{\dagger}_r \psi_2^\dagger \right), \end{aligned} \quad (15)$$

where the  $V_r^{(\prime)}$ 's are the gauge vector superfields of  $G_1(G_2)$ ,  $\psi_{1,2}$  stand for the fermionic components of  $\Phi_{1,2}$ . In more detail, we decompose  $\Phi_{1,2}$  as follows,

$$\Phi_{1,2} = \Phi_{1,2}^0 I_3 \otimes I_2 + \Phi_{1,2}^a I_3 \otimes \sigma^a + \Phi_{1,2}^\alpha t^\alpha \otimes I_2 + \Phi_{1,2}^{a\alpha} t^\alpha \otimes \sigma^a, \quad (16)$$

where  $\sigma^a$  ( $a = 1 - 3$ ) and  $t^\alpha$  ( $\alpha = 1 - 8$ ) are Pauli and Gell-Mann matrices, respectively. It is the scalar components of  $\Phi_{1,2}^0$  which get VEVs as written down in Eq. (10). It is then easy to see that the  $\psi_{1,2}^a$  combine  $\lambda_2^{(\prime)}$ , and the  $\psi_{1,2}^\alpha$  combine  $\lambda_3^{(\prime)}$  to form the massive gaugino states for the  $SU(2)$ 's and  $SU(3)$ 's; and  $\psi_{1,2}^0$  combine  $\lambda_1^{(\prime)}$  for  $U(1)$ 's, after gauge symmetry breaking. Due to Eq. (7), the higgsino  $(v_2\psi_1 + v_1\psi_2)/\sqrt{v_1^2 + v_2^2}$  and the fermionic component of  $Y$  form a massive Dirac higgsino state with mass  $c'\sqrt{v_1^2 + v_2^2}$ . Considering the soft masses, we see that the mass matrices of the gauginos and the higgsino  $(v_1\psi_1 - v_2\psi_2)/\sqrt{v_1^2 + v_2^2}$  can be written as

$$M_r = \begin{pmatrix} M_{\lambda_r} & 0 & g_r \sqrt{v_1^2 + v_2^2} \\ 0 & M_{\lambda'_r} & g'_r \sqrt{v_1^2 + v_2^2} \\ g_r \sqrt{v_1^2 + v_2^2} & g'_r \sqrt{v_1^2 + v_2^2} & 0 \end{pmatrix}. \quad (17)$$

Any mass eigenstate is a mixture of the gauginos and the higgsinos. All the gauginos are massive. As expected, if the soft masses  $M_{\lambda_r}$  and  $M_{\lambda'_r}$  are both zero, the SM gauginos are massless. Soft masses are required to make the SM gauginos massive. It is interesting to note, however, if only one of the soft gaugino mass, say  $M_{\lambda_r}$  vanishes, the SM gaugino masses are still massive. Note that we have the freedom to add a mass term for  $\Phi_1$  and

$\Phi_2$  in Eq. (7), which gives nonzero contribution to the (3-3) entry in the matrix (17). Our previous considerations will be unaffected if this mass is not too large to break the gauge symmetry.

Although the model we have described can be self-consistent, it is not complete from the GUT point of view. GUT partners of the messengers and the Higgs' should be introduced. Note that although the messengers and the Higgs' are in fundamental or bi-fundamental representations of  $G_1 \otimes G_2$ , they are not necessarily in fundamental representations of unified groups. If  $G_i$  unifies into  $SU(5)$ , they are in **10**-representation of  $SU(5)$ . On the other hand, if  $G_i$  unifies into  $SO(10)$ , they are part of the fundamental representation. For the messengers, the following partner fields are introduced,

$$\begin{aligned}
T_{1e}, T'_{1e} &= (1, 1, 2, 1, 1, 0), \quad \bar{T}_{1e}, \bar{T}'_{1e} = (1, 1, -2, 1, 1, 0); \\
T_{1u}, T'_{1u} &= (\bar{3}, 1, -\frac{4}{3}, 1, 1, 0), \quad \bar{T}_{1u}, \bar{T}'_{1u} = (3, 1, \frac{4}{3}, 1, 1, 0); \\
T_{2e}, T'_{2e} &= (1, 1, 0, 1, 1, 2), \quad \bar{T}_{2e}, \bar{T}'_{2e} = (1, 1, 0, 1, 1, -2); \\
T_{2u}, T'_{2u} &= (1, 1, 0, \bar{3}, 1, -\frac{4}{3}), \quad \bar{T}_{2u}, \bar{T}'_{2u} = (1, 1, 0, 3, 1, \frac{4}{3}).
\end{aligned} \tag{18}$$

They will be generally denoted as  $T_{GUT}$  and  $\bar{T}_{GUT}$ . Similarly for the Higgs', we introduce their GUT partners  $\Phi_{1GUT}$  and  $\Phi_{2GUT}$ . For all these fields, we trivially write down mass terms in the superpotential,

$$\mathcal{W}_3 = m_{T_{GUT}}(T_{GUT}\bar{T}_{GUT}) + m_{\Phi_{GUT}}\text{Tr}(\Phi_{1GUT}\Phi_{2GUT}). \tag{19}$$

Both  $T_{GUT}$ 's and  $\Phi_{GUT}$ 's are just GUT partner fields of the messengers and the Higgs'.  $m_{T_{GUT}}$  is about  $\sim m_j$  in Eq. (3), and  $m_{\Phi_{GUT}} \sim m_\Phi$ . These partner fields are not messengers and Higgs's themselves, because they do not play any role in SUSY breaking mediation and gauge symmetry breaking.

Numerically we consider two cases of the gauge coupling constants. Unifications in  $G_1$  and in  $G_2$  are implied, although we do not study such unifications in any detail in this paper. To be natural, the  $G_1 \times G_2$  gauge symmetry breaking scale  $v$  is required to be at (1 – 10) TeV. The first case is that  $g_r$  and  $g'_r$  are at the same order. From Eq. (13), we see that they should be close to the values of the SM gauge coupling constants at the energy scale  $v$ ,

namely  $g_r \sim g'_r \sim 0.1$ . The second case is that the  $g_r$ 's are much larger than the  $g'_r$ 's. Only the  $g'_r$ 's are close to the SM couplings,  $g_r \gg g'_r \sim 0.1$ . In any case,  $\mu_{\text{SUSY}}$  and  $m_i$ 's are taken to be about  $(100 - 1000)$  TeV. Hence messengers  $T_i$ 's have masses around  $(100 - 1000)$  TeV. The soft masses of the three generation matters are about 100 GeV. In the first case, the soft masses of the  $\Phi_{1,2}$  are about  $(100 - 1000)$  GeV. By taking  $\mu' \sim (1 - 10)$  TeV, we obtain  $v \simeq (1 - 10)$  TeV. The soft masses of the  $\Phi_{1,2}$  do not play a significant role. The gauge symmetry breaking basically determines the mass pattern. The gauginos corresponding to the broken groups, which eat the higgsinos, are of masses  $\sim g_r^{(\prime)} v \simeq (100 - 1000)$  GeV. The mass matrix (17) results in SM gaugino masses of about  $\sim M_{\lambda_r^{(\prime)}} \sim 100$  GeV.

The second case is more interesting. Because  $G_1$  is strong, the  $\Phi_{1,2}$  are as heavy as 100 TeV. Taking  $\mu' \sim 100$  TeV in Eq. (12), it is seen that through tuning, we can have  $v \sim 10$  TeV. The gauginos of  $G_1$  are about  $(10 - 100)$  TeV. The  $G_2$  gauginos which largely mix with the higgsinos are  $\sim g'_r v \sim 1$  TeV. In this case the gauginos are generally heavier than those of the first case.

Now let us discuss EWSB. A pair of Higgs superfields  $H_u$  and  $H_d$  which are nontrivial only under  $G_2$  are introduced. They are just the SM-like two Higgs doublets in  $G_2$ . The soft masses of them, similar to that of the three generation matter, are generated at the two-loop level,  $\sim 100$  GeV. After gauge symmetry breaking  $G_1 \times G_2 \rightarrow \text{SM}$ , also like the three generation matter, those Higgs doublets have the expected quantum numbers in the SM. The  $\mu$ -term and  $B_\mu$ -term are essential for EWSB. As usual, we do not assume direct interactions of the electroweak Higgs and  $X$ . They can be introduced straightforwardly in the ways discussed in models with SM gauge groups [9–11,19,20]. With the correct  $\mu$ - and  $B_\mu$ -terms, the large top quark Yukawa coupling radiatively induces EWSB.

In the second case of gauge couplings discussed above, we find that  $\Phi_1$  and  $\Phi_2$  play very useful roles in EWSB. We introduce the following nonrenormalizable interaction in the superpotential

$$\mathcal{W}_3 = c'' \frac{1}{\mu'} \text{Tr} (\Phi_1 \Phi_2) \text{Tr} (H_u H_d), \quad (20)$$



where the coupling  $c'' \sim O(1)$ . It results in that  $\mu \simeq \frac{v^2}{\mu'} \simeq 1$  TeV. Note that this superpotential does not produce a  $B_\mu$ -term at tree level. The  $B_\mu$ -term should be in the form of

$$c''' X \text{Tr} (H_u H_d) \quad (21)$$

with a very small effective coupling constant  $c''' \sim 10^{-4}$ . Then  $B_\mu \simeq c''' \mu_{\text{SUSY}}$ .  $c'''$  may be understood as originating from  $\mathcal{W}_3$  at the two loop-level, as shown in Fig. 1. From the figure we obtain

$$\frac{B_\mu}{\mu} \simeq \left( \frac{\alpha_r}{4\pi} \right)^2 \frac{1}{\mu'} \left( c_1 \frac{\mu_{\text{SUSY}}^2}{m_1} \right)^2 \sim 1 \text{ TeV}, \quad (22)$$

after taking  $\alpha_r$  to be  $O(1)$ .

In summary, we have proposed a SUSY  $SU(3)_1 \times SU(2)_1 \times U(1)_1 \times SU(3)_2 \times SU(2)_2 \times U(1)_2$  model with GMSB. The messenger fields  $T_{1,2}^{(\prime)}$  and  $T_{1,2}^{-(\prime)}$ , the Higgs fields  $\Phi_1$  and  $\Phi_2$  are in simple forms. The superpotential is given as

$$\mathcal{W} = \mathcal{W}_0 + \mathcal{W}_1 + \mathcal{W}_2 + \mathcal{W}_3. \quad (23)$$

The SM is obtained after gauge symmetry breaking. In the case that  $SU(3)_1 \times SU(2)_1 \times U(1)_1$  is strong, an EWSB scenario has been discussed. This model predicts additional gauge bosons, gauginos and Higgs particles with masses ranging from 100 GeV to 100 TeV depending on the choices of gauge coupling constants. In the interesting case of  $G_1$  being strong, the SM gaugino masses are predicted to be about 1 TeV which are generally heavier than the sfermions and higgsinos in the SM. Future experiments will check this type of models.

Several final remarks are in order. (i) The model can be extended to have unifications in  $G_1$  and in  $G_2$  separately. The unification of strong  $G_1$  avoids the Landau pole problem. It should occur at the energy scale not far above the messenger scale  $\sim 100$  TeV. However, the introduction of Higgs fields  $\Phi_{1,2}$  adds many flavors into the model. It makes both  $G_1$  and  $G_2$  being non-perturbative at 100 TeV. This non-perturbative unification is beyond the scope of this work. The values of the SM coupling constants are almost fully determined

by those of  $G_2$ . Therefore the unification of  $G_2$  will explain the observed unification of the gauge coupling constants in the minimal SUSY SM.

(ii) Nontrivial fermion mass origin can be considered in case  $G_1$  is strong. We may move the third family matter fields into  $G_1$ . A non-SUSY version of  $G_1 \times G_2$  should be studied in this case. Note that if we switch off the strong  $SU(2)$  in  $G_1$ , our model looks like a SUSY top-color model [13], but with a simpler messenger structure.

(iii) The relation of the SUSY breaking scale and the  $G_1 \times G_2$  gauge symmetry breaking scale should be studied further, especially considering that in the strong  $G_1$  case,  $\mu_{\text{SUSY}} \sim \mu'$ . We have noted that certain cancellation can be made by tuning  $\mu'$  in Eq. (12). But this is not a fine tuning. It is natural in the sense of 't Hooft. With such a tuning, a small number, namely a lower energy scale can be generated.

(iv) As having been noted after Eq. (17), if  $M_\lambda = 0$ , SM gauginos are still massive. Therefore generally speaking,  $T_1^{(\prime)}$  and  $\bar{T}_1^{(\prime)}$  fields, as well as their GUT partners are not necessary to make the models phenomenologically viable.

(v) The discussion on EWSB was not satisfactory, because it relies on complicated or non-renormalizable interactions. In the case of  $G_1$  being strong, new matter or the third family matter can be introduced in the  $G_1$  sector. Because of GMSB, the superpartners in this sector are very heavy  $\sim 100$  TeV. They decouple at  $(1 - 10)$  TeV energy scale. At this low energy scale the fermions, on the other hand, can form condensates due to the strong gauge interactions. Thus, there exists a possibility that EWSB occurs dynamically in this framework.

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# FIGURES

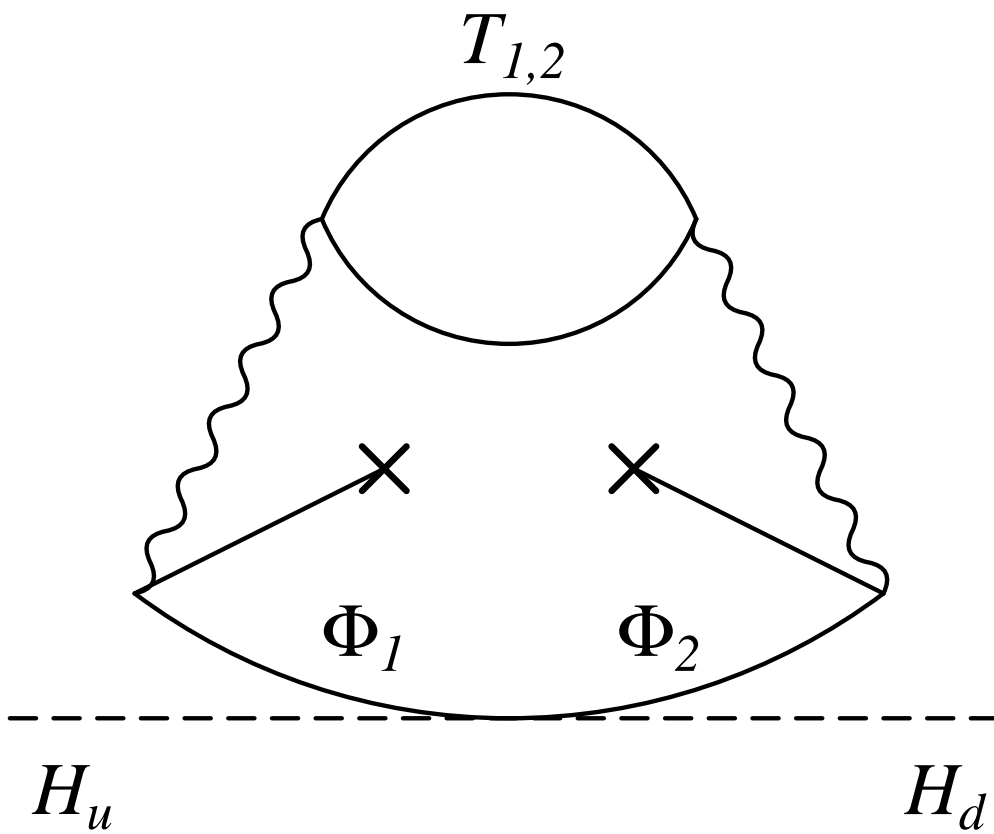


FIG. 1. *Two-loop generation of the  $B_\mu$  term.*